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170 and 180 miles—say 175. The most probable velocity therefore decidedly indicates a hyperbolic orbit.

Owing perhaps to the late hour at which the meteor appeared but few observations of the phenomenon were reported. Several letters of inquiry brought no available response, and Prof. Cleveland Abbe of the U. S. Signal Service informed me that no accounts of the meteor were received at the Washington Office.

Bloomington, Indiana, Sept., 1878.

## QUINQUESECTION OF THE CIRCUMFERENCE OF A CIRCLE.

BY PROF. L. G. BARBOUR, RICHMOND, KENTUCKY.

*Theorem.*—Let  $C$  be the centre of a circle;  $AD$ , a diameter. Divide  $AC$  in extreme and mean ratio, putting the larger segment next the centre. Then from  $D$  as a centre, with a radius equal to  $DB$ , describe an arc cutting the circumference in  $F$ .

The arc  $AF$  will be one fifth of the circumference.

*Demonstration.* — Join  $BF$ ,  $CF$  and  $DF$  and draw  $CE$  parallel to  $BF$

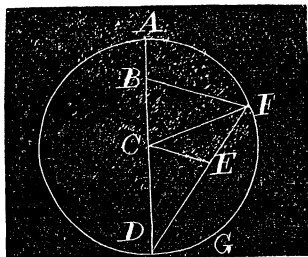
By hypothesis

$$AB : BC :: BC : AC.$$

By composition, because  $AC = CD$ ,

$$CD : BC :: BD : CD;$$

$$\therefore BC : CD :: CD : BD.$$



The triangles  $BCF$  and  $CFD$ , having the same altitude, are to each other as their bases;  $\therefore BFC : CFD :: BC : CD$ . Similarly

$$CFD : BFD :: CD : BD.$$

But the last couplets of these proportions, themselves form a proportion;

$$\therefore BFC : CFD :: CFD : BFD.$$

Also, since  $CE$  is parallel to  $BF$ , we have

$$DCE : CFD :: CFD : BFD.$$

Comparing this with the last proportion, we find  $BFC = DCE$ . These two triangles then are equal in area; the base  $DE$  of the one, is equal to the base  $CF$  of the other, for  $CED$  is isosceles, because similar to  $BFD$ , and hence  $DE = CD$ . Moreover, the angle  $DCE$  opposite  $DE$  is equal to the angle  $DBF$ . But any two triangles of equal areas, equal bases, and having the angles opposite the bases, equal, each to each, are equal in all

their parts.  $\therefore$  angle  $BFC$  = angle at  $D$ .  $BCF = CBF = DCE$ . But  $BCF$ , being measured by the arc  $AF$ , is double the angle at  $D$ , which is measured by half the arc  $AF$ ;  $\therefore \angle BCF = 2 \angle BFC$ .

The angles  $BFC$ ,  $BCF$  and  $CBF$  are together equal to two right angles,  $\therefore BCF = \frac{2}{3}$  of two right angles, or  $\frac{1}{3}$  of four right angles;  $\therefore AF$  is  $\frac{1}{3}$  of the circumference. Q. E. D.

SCHOLIA.—1. Taking the angle  $D$  as the unit of measurement, we have  $D = BFC = FCE = CFE$ ,  $2D = FBC = FCB = BFD = DCE = DEC$ ;  $3D = CEF = BCE = FCD = FBA$ .

2.  $DE = DC = CF = BF =$  radius;  $BC = CE = EF =$  greater segment of radius. Also the triangles  $CDE$ ,  $CEF$ ,  $DCF$ ,  $BCF$  and  $BFD$ , i. e., all the triangles in the figure, are isosceles.

3. Laying off from  $F$  towards  $D$  an arc equal to  $AF$ , we get a remainder  $DG$  which is  $\frac{1}{10}$  of the circumference. We know from Euclid that the chord from  $D$  to  $G$  is equal to  $BC$ ; hence  $BC$ ,  $CE$  and  $EF$  are each equal to the chord of a decagon, and  $DE$ , &c., to the chord of a hexagon. Also,  $BD = DF =$  chord of  $\frac{2}{3}$  of two right angles, or  $\frac{2}{3}$  of one right angle,  $\therefore FCD = FBA$  &c. = angle between two consecutive sides of a regular pentagon.

4. Since  $BC : CD :: CD : BD$ , chord of a regular inscribed decagon : chord of hexagon :: chord of hexagon : chord of  $108^\circ$ .

Putting  $CD =$  radius  $= 1$ , chord  $36^\circ = \frac{1}{\text{chord } 108^\circ}$ ;  $\therefore \sin 18^\circ = \frac{1}{\sin 54^\circ}$ .

Since  $BD = CD + BC$ , chord  $108^\circ = 1 + \text{chord } 36^\circ$ .  $\therefore \sin 54^\circ = .5 + \sin 18^\circ$ , as may be seen in a table of natural sines.

To find the length of  $BC$ , which we will call  $x$ ,  $x : 1 :: 1 : x + 1$ ;  $\therefore x^2 + x = 1$ , whence  $x = -\frac{1}{2} \pm \frac{1}{2}\sqrt{5} = BC$ ;  $\therefore BD = DF = +\frac{1}{2} + \frac{1}{2}\sqrt{5}$ .

$AF^2 = AD^2 - DF^2 = 4 - \frac{1}{4} - \frac{1}{2}\sqrt{5} - \frac{5}{4} = \frac{5}{2} - \frac{1}{2}\sqrt{5} = BC^2 + CD^2$ ;  $\therefore$  the side of a regular inscribed pentagon is the hypotenuse of a right angled triangle, the other sides being the side of a regular inscribed hexagon and that of a regular inscribed decagon. This has been noticed by Young.

## A PROBLEM AND ITS SOLUTION.

BY DR. H. EGGERS, MILWAUKEE, WISCONSIN.

*Problem.* — Given in a plane three fixed right lines,  $L_1, L_2, L_3$ , and in each of them a fixed point, respectively,  $A_1, A_2, A_3$ : required a right line  $M$ , which shall cut off on  $L_1, L_2, L_3$ , three equal distances counting from